1. (11 points) A firm sells its product in a perfectly competitive market where other firms charge a price of $40 per unit. The firm’s total costs are \( C(Q) = 40 + 8Q - 2Q^2 + Q^3 \).

(a) How much output should the firm produce in the short run?
Set \( P = MC \) to get \( 40 = 8 - 4Q + 3Q^2 \). Solve for \( Q \) to get \( Q = 4 \) units.

(b) What price should the firm charge in the short run?
$40.

(c) What are the firm’s short-run profits?
Revenues are \( R = (40)(4) = $160 \), costs are \( C = 40 + 8(4) - 2(4)^2 + (4)^3 = $104 \), so profits are $56.

(d) What adjustments should be anticipated in the long run?
Entry will occur, the market price will fall, and the firm should plan to reduce its output. In the long-run, economic profits will shrink to zero.

(e) Draw average variable cost and marginal cost for \( Q \) between 0 and 5 and draw average fixed cost and average total cost for \( Q \) between 1 and 5. Draw all curves in the same figure and as exactly as possible. How can you determine the long run market price of the firm’s product graphically?
Because other firms will copy your production process in the long run, there will be market entry until profits are zero, and \( MC = ATC \).
2. (5 points) The accompanying graph summarizes the demand and costs for a firm that operates in a monopolistically competitive market.

(a) What is the firm’s optimal output?
7 units.

(b) What is the firm’s optimal price?
$130.

(c) What are the firm’s maximum profits?
$140, since ($130 - 110) x 7 = $140.

(d) What adjustments should the manager be anticipating?
This firm’s demand will decrease over time as new firms enter the market. In the long-run, economic profits will shrink to zero.

3. (10 points) You are the manager of a monopoly, and your demand and
cost functions are given by $P = 56 - 2Q$ and $C(Q) = 40 + 8Q - 2Q^2 + Q^3$, respectively.

(a) What price-quantity combination maximizes your firm’s profits?
$MR = 56 - 4Q$ and $MC = 8 - 4Q + 3Q^2$. Setting $MR = MC$ yields $Q = 4$. Thus, $P = 56 - 8 = 48$.

(b) Calculate the maximum profits.
Revenues are $R = ($4)(48) = $192 and costs are $C(4) = 104$, so the firms profits are $88$.

(c) Is demand elastic, inelastic, or unit elastic at the profit-maximizing price-quantity combination?
At the profit-maximizing quantity, $MR > 0$. For this, demand has to be elastic.

(d) What price-quantity combination maximizes revenue?
TR is maximized when $MR = 0$. Setting $MR = 0$ yields $Q = 14$. The price at this output is $P = 56 - 2(14) = $28$.

(e) Calculate the maximum revenues.
Using the results from part d, the firms maximum revenues are $R = ($28)(14) = $392$.

(f) Is demand elastic, inelastic, or unit elastic at the revenue-maximizing price-quantity combination?
At the revenue-maximizing quantity, $MR = 0$. For this, demand has to be unit elastic.

4. (3 points) The elasticity of demand for a firm’s product is $-2.5$ and its advertising elasticity of demand is 0.25.

(a) Determine the firm’s optimal advertising-to-sales ratio.
The optimal advertising to sales ratio is given by $\frac{A}{R} = \frac{E_{Q,A}}{E_{Q,P}} = 0.25 \Rightarrow \frac{A}{R} = 0.1$.

(b) If the firm’s revenues are $40,000, what is its profit-maximizing level of advertising?
$\frac{A}{R} = \frac{E_{Q,A}}{E_{Q,P}} \Rightarrow \frac{A}{40000} = 0.1 \Rightarrow A = $4000$

5. (6 points) A monopolist’s inverse demand function is $P = 200 - 3Q$.
The company produces output at two facilities; the marginal cost of producing at facility 1 is $MC_1(Q_1) = 4Q_1$, and the marginal cost of producing at facility 2 is $MC_2(Q_2) = 2(Q_2)^2$.

(a) Provide the equation for the monopolist’s marginal revenue function.
Since the inverse demand is $P = 200 - 3Q_1 - 3Q_2$, marginal revenue is $MR(Q) = 200 - 6Q_1 - 6Q_2$.

Using $Q, Q_1, Q_2$ correctly: $1 \text{ pt}$
(b) Determine the profit-maximizing level of output for each facility.

Solving the equations

\[
200 - 6Q_1 - 6Q_2 = 4Q_1 \\
200 - 6Q_1 - 6Q_2 = 2(Q_2)^2
\]

simultaneously yields \( Q_1 = 16.55 \) and \( Q_2 = 5.75 \).

(c) Determine the profit-maximizing price.

\[ P = 200 - 3Q_1 - 3Q_2 = \$133.1 \]

6. (4 points) The elasticity of demand for the product of a monopoly is estimated to be \(-5\) and constant. The firm’s marginal cost is constant at \$12 per unit.

(a) Express the firm’s marginal revenue as a function of its price.

Based on a price elasticity of demand of \(-5\), the monopolists marginal revenue is

\[ MR = P \left(1 - \frac{1}{5}\right) = \frac{4}{5}P \]

(b) Determine the profit-maximizing price.

Since the monopolist maximizes profits where \( MR = MC \), the profit-maximizing price can be obtained by solving the following equation: \( \frac{4}{5}P = 12 \), or \( P = \$15 \).

The Number of Firms in Monopolistic Competition

If the representative consumer has utility \( U = X^\beta Y^{1-\beta} \), the demand for \( X \) is \( \beta M/p_c \). Assume that \( X \) is produced from \( n \) varieties \( X_i \):

\[ X = \left[ \sum_{i=1}^{n} X_i^\alpha \right]^{1/\alpha} \]

If the prices of \( X_i \) are \( p_i \), the cheapest way of producing one unit of \( X \) costs

\[ p_c = \left[ \sum_{i=1}^{n} p_i^{1-\sigma} \right]^{1/(1-\sigma)} \]

with \( \sigma = \frac{1}{1-\alpha} \).

NF1 (5 points) Assume that all firms \( i = 1 \ldots n \) sell the same amount \( X_i = X \) at the same price \( p_i = p \). Express demand for \( X \) as a function of \( n \) and \( p \). How high does \( X \) have to be so that this amount of \( X \) is reached?

The above equations simplify to

\[ X = [nX^\alpha]^{1/\alpha} = n^{1/\alpha}X \]

and

\[ p_c = [np^{1-\sigma}]^{1/(1-\sigma)} = n^{1/(1-\sigma)}p \]

1 pt
Demand for $X_c$ becomes

$$X_c = \frac{\beta M}{pn^{1/(1-\sigma)}}$$

and thus

$$X = \frac{X_c n^{1/\alpha}}{n} = \frac{\beta M}{p n^{1/(1-\sigma)} n^{1/\alpha}} = \frac{\beta M}{pn^{1/(1-\sigma)\frac{n}{\sigma}}(\sigma-1)} = \frac{\beta M}{pn}.$$  

**NF2** (5 points) Now assume that all $n$ firms produce $X_i$ with the same fixed and marginal costs $F$ and $c$. If there are a lot of firms, the single firms takes the other firms pricing and supply decisions as given. Faced with iselastic demand for their own product, the profit maximizing price that this firm sets is (without proof)

$$p \left(1 - \frac{1}{\sigma}\right) = c.$$  

However, firms will enter or exit the market until the entry of one additional firm would make profits negative. Assume the special case, where the entry of the last firm caused profits to be exactly zero. Write down the zero profit condition for the price and solve together with the above equation for $p$ and $X$.

From the given equation, we conclude

$$p = \frac{\sigma c}{\sigma - 1}.$$  

The zero profit condition reads

$$p = c + F/X,$$

and we get

$$X = \frac{F}{p - c} = \frac{F}{\frac{\sigma c}{\sigma - 1} - c} = \frac{F}{\frac{\sigma c}{\sigma - 1} - c} = \frac{(\sigma - 1)F}{c}.$$  

Our analysis says that firms will enter until the entry of one more firm would cause $X < \frac{(\sigma - 1)F}{c}$, and thus $p > c + F/X$. 

**5** should say '<...
NF3 (4 points) Use the above results to determine the number of firms that will enter this market!

So far, we have

\[
X = \frac{(\sigma - 1)F}{c} \quad \text{and} \quad X = \frac{\beta M}{\rho n}. \quad 1 \text{ pt}
\]

Inserting one into the other and solving for \( n \) gives

\[
N = \frac{c\beta M}{p(\sigma - 1)F} = \frac{\beta M}{\sigma F}. \quad \text{result without p in it: 1 pt}
\]

NF4 (For the interested. 0 points) Assume the general case: firms have entered until there is some small but positive profit, which would become negative if one more firm entered the market. What is the precise expression for \( n \) now? And how would you express the profits of a single firm?

In our situation, \( n \) implies \( p > c + F/X \) and \( n + 1 \) firms would imply \( p < c + F/X \). Therefore, \( \beta M/(\sigma F) \), which solved \( p = c + F/X \) lies between \( n \) and \( n + 1 \). Thus, we can say that

\[
n = \frac{c\beta M}{p(\sigma - 1)F} = \left\lfloor \frac{\beta M}{\sigma F} \right\rfloor \quad 0 \text{ pt}
\]

The profits of the single firm are then

\[
\pi = X(p - c) - F
\]
\[
= \frac{\beta M}{\frac{\sigma}{\sigma - 1}n} \left( \frac{\sigma c}{\sigma - 1} - c \right) - F
\]
\[
= \frac{\beta M}{n} \left( 1 - \frac{\sigma - 1}{\sigma} \right) - F
\]
\[
= \frac{\beta M}{n\sigma} - F
\]
\[
= F \left( \frac{\beta M}{\frac{\sigma F}{\beta M}} - 1 \right). \quad 0 \text{ pt}
\]

Note: Because \( \frac{\beta M}{\sigma F} < n + 1 \), we can say that

\[
\pi < F \left( \frac{n + 1}{n} - 1 \right) = \frac{F}{n}.
\]